Proposed content course on “Local convergence in random graphs”

(based on Random Graphs and Complex Networks Volume 2)

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In this document, we propose a course content for a course on local convergence in random graphs, based on Random Graphs and Complex Networks Volume 2.

(1) Start with a recap of the random graph models to be covered in this course, by discussing some of their properties in [II, Chapter 1]. Particularly [II, Section 1.3] is then convenient. Should you wish to brush up more details on these models, you can discuss the basics in [I, Chapter 6] for the generalised random graph, [I, Chapter 7] for the configuration model, and [I, Chapter 8] for the preferential attachment model.

(2) Discuss the theory of local convergence in [II, Chapter 2]. Cover the basic notation of rooted graphs and their metric properties in [II, Section 2.2], the notion of local convergence of deterministic graphs in [II, Section 2.3], and that of random graphs in [II, Section 2.4].

(3) Continue with consequences of local convergence in [II, Section 2.5]. Here, you can pick those consequences that you like best.

(3b) If time permits, you can discuss the relation between the size of the giant and the local limit in [II, Section 2.6]. This is useful as intuition, and the proof is relatively straightforward.

(4) Discuss local convergence of inhomogeneous random graphs in [II, Section 3.5]. Here, you can either discuss inhomogeneous random graphs in full generality by covering [II, Sections 3.2–3.4], or instead focus on the generalised random graph in [II, Section 3.5.3], combined with [II, Section 3.5.4], whose proof will be given in the next chapter.

(5) Discuss local convergence of configuration models in [II, Section 4.2]. You can choose to skip the analysis for uniform random graphs with prescribed degrees in [II, Section 4.2.3] and [II, Section 4.2.4].

(5b) You can choose to discuss the size of the giant in configuration models in [II, Section 4.3], with or without proof. There are two proofs, a ‘giant is almost local proof in [II, Section 4.3.1], and a continuous-time exploration proof in [II, Section 4.3.2].

(6) Discuss local convergence of preferential attachment models in [II, Section 5.3]. This proof consists of two steps. In the first, a Pólya urn description is given for preferential attachment models in [II, Section 5.3.3]. This description is essential in the proof, and it is useful to first cover the discussion of exchangeable random variables and De Finetti’s Theorem in [II, Section 5.2]. The second step contains the analysis of the Pólya urn description in [II, Section 5.4]. This part is quite technical, and it might be a good idea to explain this at a heuristic level, for example by using [II, Lemma 5.17].

(7) It may be interesting to close the course with an informal description of the small-world properties of the random graph models discussed so far. [II, Section 6.2] discusses small-world properties of inhomogeneous random graphs, [II, Section 7.2] of configuration models, and [II, Section 8.3] of preferential attachment models. [II, Chapters 6 and 7] offer possibilities to perform small partial proofs. The lower bounds on distances are the simplest, and can be found in [II, Section 6.3] for the generalised random graph, and [II, Section 7.3.2] for the configuration model. For a glimpse of the reason behind ‘ultra-small’ distances, it is useful to check the short and sweet argument in [II, Section 7.3.4] for the configuration model.